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Size-Dependent Gender Gaps in Entrepreneurship: The Case of Chile

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Abstract

This paper documents differences in firm size depending on whether their manager is a man or a woman and studies the aggregate implications of these gender gaps in Chile. We document that in 2007 less than a quarter of firms are managed by women and that this gap takes its largest value for managers with tertiary education or more. In terms of their number of workers, female-run firms are on average about three times smaller than those run by men. Moreover, the ratio of men to women managers is always above one, but it is much higher for large and medium firms than for small or micro ones. These differences remain significant after controlling for several manager and firm characteristics. We then use an extended version of the theoretical framework developed in Cuberes and Teignier (2016) to incorporate these facts and obtain quantitative predictions about their effects on aggregate productivity and income in Chile. We find that the observed gender gaps in entrepreneurship in Chile generate a fall in aggregate productivity and aggregate income of 7.5%.

Keywords: gender inequality; aggregate productivity; firm size; talent misallocation, Chile JEL Codes: E2, J21, J24, O40

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Introduction

Gender inequality is present in many socioeconomic indicators around the world in both developed and developing countries. These gaps can be observed in several dimensions: education, earnings, occupation, access to productive inputs, political representation, or bargaining power inside the household.[1] One important aspect of gender inequality in the labor market that has not been much studied in the literature is the low presence of women in entrepreneurial activities. Looking at data on employers and self-employment, we observe that women are underrepresented in most countries. In this paper we focus on Chile, where the share of employers in the working-age population in 2010 is 1.5% for males and 0.6% in the case of females and the share of self-employed is 8.8% for males and 4.5% for females. We start the analysis by providing some descriptive statistics on the degree of gender inequality in entrepreneurship in Chile and, next, we develop a quantitative macroeconomic model to compute the aggregate effects of these gender gaps.

The data show that women are clearly underrepresented in entrepreneurship. We explore differences in several firm characteristics depending on whether these firms are run by men or women. First we consider different measures of firm size and, then, we use a simple regression analysis to assess the robustness of the correlation between firm size and the manager's gender. We then explore whether the existing gender gaps in entrepreneurship vary by education in order to shed light on the possible explanations.

We find that only about one fifth of the firms are run by women and this gap takes its largest value for managers with tertiary education or more. Moreover, female-run firms are significantly smaller than the male-run ones, even after controlling for the age, education, and experience of the manager as well as the firm sector and the type of firm in terms of ownership (e.g. public vs private). The average number of employees of female-run firms is 6, while the average of employees of male-run firms is 18. The ratio of men to female managers is 56 for the largest firms and around three for the smallest ones.

We next extend the theoretical framework developed in Cuberes and Teignier (2016) to incorporate these facts, with the final goal of quantifying the aggregate consequences of these firm-level gender gaps on aggregate productivity and income. We use a general equilibrium occupational choice model where agents are endowed with a random entrepreneurship skill, based on which they decide to work as either employers, self-employed, or workers. An employer in this model produces goods using a span-of-control technology that combines his or her entrepreneurship skills, capital, and workers. This span-of-control element implies that more talented agents run larger firms than less talented agents, as in Lucas (1978). On the other hand, a self-employed agent can produce goods using a similar technology - adjusted by a productivity parameter - but without hiring any workers.

The model assumes that men and women are identical in terms of their managerial skills. However, women are subject to exogenous frictions in the labor market, implying that a fraction of women who would like to be employers or self-employed are excluded from these occupations. These restrictions distort the occupational allocation and reduce aggregate productivity and income per capita. The intuition behind the output loss is as follows. When a woman with high management skills does not become an employer, a less skilled man will take her position and become the manager of a firm and, as a consequence, output per worker will fall. An important question not previously addressed is which type of women are excluded from entrepreneurship. In our framework, if all women faced the same probability of not participating in entrepreneurship, we find an income loss of 5.2%, but once we make more talented women less likely to participate in entrepreneurship, we quantify a significantly larger fall in productivity and income equal to 7.5%. The loss is larger in the second case because the average talent of both employers and self-employed falls more.

1 See the World Development Report 2012 (World Bank, 2012) for a comprehensive review of these and other gender gaps.

There are now a few articles studying the macroeconomic effects of gender gaps in the labor market.[2] The International Labor Organization provides some estimates of the output costs associated with labor gender gaps in the Middle East and Northern Africa but without proposing any specific theoretical model (ILO, 2014). Cavalcanti and Tavares (2016) construct a growth model based on Galor and Weil (1996) in which there is exogenous wage discrimination against women. Calibrating their model using U.S. data, they find very large effects associated with these wage gaps: a 50 percent increase in the gender wage gap in their model leads to a decrease in income per capita of a quarter of the original output. Hsieh et al. (2019) use a Roy model to estimate the effect of the changing occupational allocation of white women, black men, and black women between 1960 and 2008 on U.S. economic growth and find that the improved allocation of talent within the United States accounts for 17 to 20 percent of growth over this period.

With respect to the gender gaps in entrepreneurship and their aggregate effects, Cuberes and Teignier (2016) calculate the macroeconomic effects of gender inequality in the labor market using data from the International Labor Organization for a large sample of countries, but they do not explore differences in firm characteristics by gender. More recently, Bento (2021) shows that female entrepreneurship has risen in the U.S. since 1982 and concludes that this change accounts form more than 12% of the output growth in that country. Chiplunkar and Goldberg (2021) develop a framework to identify and quantify barriers to entry and operation faced by female entrepreneurs in developing countries, and apply it to the Indian economy. Ranasinghe (2021) uses a standard model for measuring misallocation and finds substantial differences across male and female establishments in a large sample of low- and middle-income countries. Morazzoni and Sy (2021) study the effects of gender gaps in credit access in a model with financial frictions. Finally, Bento et al. (2021) study the relationship between gender asymmetries in time devoted to household production and the gender gaps in entrepreneurship, firm productivity, and firm-size distribution. We see our model as a complement to these papers, which use different theoretical models than ours.

The rest of the paper is organized as follows. Section 2 describes the Chilean dataset. Section 3 sketches the general equilibrium occupational choice model based on the one in Cuberes and Teignier (2016). The numerical results of the paper are presented and discussed in Section 4 and, finally, Section 5 concludes the paper. Some more empirical results are presented in Appendices A and B, and Appendix C explains the details of the theoretical model.

Chilean data

The data used in this paper comes from the Encuestas Longitudinales de Empresas in Chile. This is a detailed dataset of firms' characteristics in Chile for the years 2007, 2008 and 2009. This version of the paper uses only the year 2007. The main goal of the survey was to characterize Chilean firms by size and sector and identify the main determinants of entrepreneurship in this country. For the first wave (2007), the survey includes formal firms that operate within the country and with sales of more than 0,1 UF in the year 2007.[3] The sample is stratified by economic sector and by size to make the results as comparable as possible. This paper complements the detailed study of gender dimensions of Chilean entrepreneurship in Arellano and Peralta (2015), which analyzes the Chilean data in the year 2009. One important difference between our study and theirs is that our discussion is interpreted using the theoretical framework in Cuberes and Teignier (2016), whereas Arellano and Peralta offer only a descriptive analysis of the data.

Since this dataset is based on survey data we need to weigh the data using expanding factors. Using these weights allows us to use make the sample representative of the universe of Chilean

2 See Cuberes and Teignier (2014) for a critical literature review of the two-directional link between gender inequality and economic growth.

3 UF (valores de la Unidad de Fomento) is a unit of account used in Chile. The figures in the paper correspond to UF's in the year 2007.

firms.[4] In particular in all that follows, we use the expanding factor "factor empresas" (expansion factor by the number of firms) to do so.[5]

Firm characteristics by gender

Firm size distribution

The sample consists of 10,213 observations. Of those, we keep those individuals who claim to be the manager ("gerente") of the firm (93.5% of them). Of the remaining 9,554 individuals, 7,623 (79.8%) of them are men and 1,931 (20.2%) are women. In all the analysis that follows, we use weighted variables. There are three indicators of firm size in the dataset: *taman˜o4*, *taman˜o6*, and *trabajadores*. *Taman˜o4* classifies firms in four categories: microempresa, pequen˜a, mediana, and grande. *Taman˜o6* gives us some more detail and includes 6 categories: microempresa 1 (between 0.1 UF and 800 UF) , microempresa 2 (between 800.1 UF and 2,400 UF), pequen˜a empresa 1 (between 2400.1 UF and 5,000 UF), pequen˜a empresa 2 (between 5,000.1 UF and 25,000 UF), mediana empresa (between 25,000.1, and 100,000 UF), and empresa grande (more than 100,000 UF). For the sake of brevity, in the paper we will present results only for the variables *taman˜o6* and *trabajadores*.

The following table shows the different categories of *tamaño6* along with the number and percentage of firms by gender in each of them.[6] This table shows significant gender differences in firms' ownership at each firm size. For example, the ratio of female to men entrepreneurs for firms of the type *Micro 2* is 0.22. Interestingly, the table shows that women are increasingly underrepresented as firm size increases.

Table 1. Number of managers by firm size and gender

	Total	Men	Women	Ratio Women/Men
Micro 1	6,376	4,692	1,483	0.32
Micro2	1,484	1,236	275	0.22
Small 1	668	627	89	0.14
Small 2	729	740	70	0.09
Medium	199	217	11	0.05
Large	97	112	2	0.02
Total	9,554	7,623	1,931	0.25

We next use the number of employees per firm (the variable *trabajadores*) as a measure of firm size. Using this measure the average firm in the sample has 16 workers. The average firm size for men is 18 workers while female-run firms only have 6 workers on average. In Figure 2, we classify firms in seven categories based on their number of workers and then calculate the gender gap in managers for each firm size. As it is apparent, the gap increases with firm size, moving from a 56% gap for the smallest firms up to an almost 90% gap for firms with the largest number of workers.

4 We use the stata command *aweights* which calculates analytic weights, i.e. weights that are inversely proportional to the vari- ance of an observation. Using other weights does not change our results in any important way. For more details on the expansion factors in this dataset, see "Manual de uso ELE1" (http://www.economia.gob.cl/estudios-y-encuestas/encuestas/encuestas-de- emprendimiento-y-empresas/primera-encuesta-longitudinal-de-empresas-ele).

5 We have chosen to weight our regressions using analytical weights too. Not weighting the regressions offers qualitatively similar results in most cases although some of the innovation regressions change substantially. For a discussion on whether to apply weights on regressions, see Cameron and Trivedi (2005).

6 Note that the sum of the number of men and women does not exactly equal the total number in the second column due to the weights applied to each observation in the sample.

Figure 1. Firm size and gender gap in the number of workers

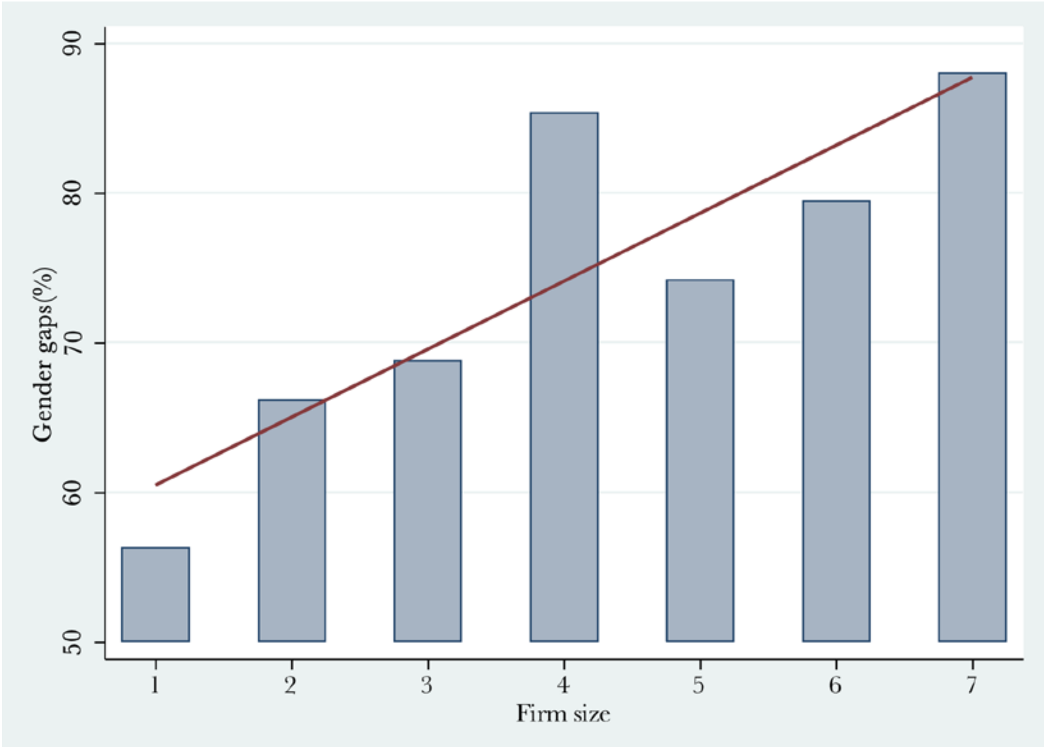


Table 1 and the numbers reported for the variable *trabajadores* confirm our previous conclusions and those of Arellano and Peralta (2015): in this sample, men entrepreneurs manage much larger firms than women. It is particularly striking that the ratio of male to female managers increases with firm size. This difference in firm size by gender is one of the main findings of this paper. Our objective in what follows is to determine whether this is still the case after we control for several firm characteristics and, in the next section, to calculate the aggregate implications of this gap using the logic of the model by Cuberes and Teignier (2016).

Regression analysis

In this subsection we aim to shed some light on the determinants of firm size in Chile as a function of characteristics of the manager and of the firm. In particular, we explore whether the very large observed differences in firm size between men and women disappear once we control for several firm and manager characteristics. We are upfront in acknowledging that these regressions only reflect correlation and, due to the many possible endogeneity issues, including selection issues and problems of omitted variables, should not be interpreted as identifying any causal effect. While the literature on the determinants of firm growth (e.g. Variyam and Kraybill, 1992) has highlighted several factors that contribute to a firm's size, our choice of covariates, described in Table 2, is very much restricted by the data that we have available.

Our main objective is to establish whether once one controls for several of these firm characteristics, the variable *Men*, a dummy variable that takes a value of 1 if the manager of the firm is a man and 0 if she is a woman, has a positive sign (so that firms managed by men are larger than those run by women) and remains statistically significant.

Table 2. Correlates in firms' regressions

	Mean	Std Dev
Men	0.66	0.47
Age	51.3	12.6

Table 2 (continued). Correlates in firms' regressions

	Mean	Std Dev
Experience	18.6	12.7
Private domestic firm	0.996	0.06
Private foreign firm	0.006	0.07
Public firm	0.001	0.03
Persona natural	0.77	0.42
Limited liability company	0.16	0.36
Individual limited liability corporation	0.01	0.12
General partnership	0.002	0.04
Closed stock company (S.A cerrada)	0.04	0.2
Open stock company (S.A abierta)	0.001	0.04
Cooperative	0.0007	0.03

Note: The dummy variable for men takes a value of 1 for men and 0 for women. All the other dummies take a value of one if the firm satisfies the characteristic specified in the first column, and 0 otherwise.

The educational dummies are constructed based on the number of years of education of the firms manager and include the following categories: no formal education, primary education, medium education or humanities, medium education TP, technical education, professional institute, university education, and postgraduate education.[7] The data contain information on the following sectors: agriculture and farming, mining, manufacturing, electricity, gas, and water, construction, commerce, hotels and restaurants, transportation, storage and communication, real estate, finance, and others.

We first explore the determinants of firms using as our dependent variable the categorical proxy *tamaño6*. As it is well-known in the literature, when the dependent variable takes a number of discrete values. The most appropriate method of estimation is an ordered choice model (Greene, 2012, p. 784), and the most commonly used ordered choice model is the ordered probit model.[8] The results of this estimation are displayed in Table 3.[9]

We estimate by ordinary least squares the following regression:

$$Firm\ Size = \alpha + \beta\_1\ Men + \gamma' X + \varepsilon$$

where *Firm Size* is measured with the variable *trabajadores*, the number of employees working in a firm and X includes the regressors discussed above.

The results of estimating this ordered probit model show that the men dummy variable is always significant at the one percent level and enters with a positive sign, indicating that firms managed by men are larger than those run by women. This is the case in specification (1) where we do not control for any other variable. Introducing education dummies (column 2) does not change this coefficient much. In specification (3) we see that the age of the manager has a nega-

7 See the MANUAL DE USO Encuesta Longitudinal de Empresas-2007 for more information on these categories.

8 An alternative would be to use OLS, which gives very similar qualitative results.

9 The coefficients displayed in this table are not the marginal effects of the regressors on the dependent variable. These effects can easily be calculated, but we do not report them here for the sake of brevity.

tive impact on firm size and that his or her experience has a positive impact on the size of the firm managed. These results seem reasonable since age and experience tend to be positively correlated and older - or more experienced managers are likely to manage bigger firms. Including these two variables reduces the size of the men coefficient but it still remains positive and highly significant. Specification (4) adds sector dummies to account for possible differences across different production sectors. Doing so does not alter the size nor the significance of the gender dummy although age is no longer statistically significant. Finally, in specification (5) we control for whether firms are domestic or foreign as well as whether they are public or private. Adding these controls does not eliminate the effect of gender and age and experience remain significant and with the same signs as in specification (2).[10] While establishing causality is a much more difficult task, these results show that the partial correlation between gender and firm size seems to be very robust.

Table 3. Determinants of firms' size using *tamaño6*

	(1)	(2)	(3)	(4)	(5)
Men	0.49***	0.44***	0.37***	0.30***	0.21***
Age	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Experience			-0.003*	-0.002	-0.005***
			(0.002)	(0.002)	(0.002)
			0.01***	0.01***	0.01***
			(0.002)	(0.002)	(0.002)
Education dummies	No	Yes	Yes	Yes	Yes
Sector dummies	No	No	No	Yes	Yes
Type of firm	No	No	No	No	Yes
Pseudo R^2	0.02	0.05	0.05	0.08	0.14
N	9,554	9,554	9,554	9,554	9,554

Table 4 shows estimates of (1) using the number of workers (*trabajadores*) as a proxy of firm size. As before, the most important result for the purposes of this paper is that the coefficient associated with the dummy variable sex is positive and highly significant in all specifications, indicating that the finding that women tend to run smaller firms than men is very robust. In this regression, the age of the manager consistently enters with a negative sign, while experience again has a positive effect.

Analysis by sector

In the previous regressions some of the sector dummies turned up to be statistically significant. Here we explore this in more detail by estimating each of the previous regressions for each sector. We estimate our regressions using the specification that has all the controls discussed above. For the sake of brevity, Table 5 reports only the coefficients associated with the gender dummy. The second column in that table also shows the percentage of women that are managers in any given sector. This figure varies wildly, with hotels and restaurants having around 55.4% of female managers and construction, which has only about 7.44% of female managers. The table shows that, in terms of size (measured by the number of employees), men-led firms are larger in manufacturing, construction, hotels and restaurants, and transportation, storage and communication.

10 The results are even stronger when we exclude the 5% smallest firms, as Table 9 in Appendix A shows.

Table 4. Determinants of firms' size using *log trabajadores*

	(1)	(2)	(3)	(4)	(5)
Men	0.46***	0.44***	0.37***	0.30***	0.21***
Age	(0.07)	(0.04)	(0.04)	(0.04)	(0.04)
Experience			-0.003*	-0.002	-0.005***
			(0.002)	(0.002)	(0.002)
			0.01***	0.01***	0.01***
			(0.002)	(0.002)	(0.002)
Education dummies	No	Yes	Yes	Yes	Yes
Sector dummies	No	No	Yes	Yes	Yes
Type of firm	No	No	No	Yes	Yes
Pseudo R^2	0.02	0.07	0.07	0.15	0.23
N	4,716	4,716	4,712	4,712	4,712

Table 5. The effect of gender on firms' characteristics by sector

	Women Emp.	Size (log trab.)
Agriculture and farming	16.4	0.04 (0.31)
Mining	10.3	0.46*** (0.25)
Manufacturing	33.1	0.41*** (0.12)
Electricity, gas, water	36.1	0.21 (0.21)
Construction	7.4	0.57** (0.23)
Commerce	41.4	0.009 (0.12)
Hotels and restaurants	55.4	0.42*** (0.13)
Transportation, storage and comm.	20.1	0.25** (0.11)
Real estate	14.6	-0.28 (0.36)
Finance	27	0.004 (0.24)
Others	49.7	0.26 (0.20)

Employers gender ratios at different education levels

In this section we examine how entrepreneurship gender ratios (defined as the number of female managers over men managers) vary at different educational levels. Table 6 shows that there are gender gaps in employers at the lowest possible level of education (0.42), but these ratios are much lower at the highest levels of education (0.31 in university education and 0.2 in postgraduate education). One possible explanation for this pattern is that the supply of educated men and women varies for men and women at different levels of education. However, national level from the Ministry of Education of Chile in 2007 and 2008 shows that the number of men and women is very similar at each education level.[11] In the next subsection we provide alternative explanations to this finding.

Table 6. Employers gender ratios by education

	Ratio Women/Men
No formal education	0.42
Primary education	0.53
Educacion media CH o humanidades	0.68
Educacion media TP	0.52
Centro de formacion tecnica Construction	0.63
Instituto profesional	0.64
College education	0.31
Postgraduate education	0.2
Total	0.52

Discussion of empirical findings

All in all, we find that the entrepreneurship gap is positive and increasing with firm size. Moreover, this gap tends to increase with the education level of the firm manager. A possible interpretation of these facts is that the implicit barriers faced by women to become entrepreneurs are larger for more skilled women than for less skilled ones.[1212] One can think of many different explanations for this, both from the demand and the supply side. From the demand side, these barriers could be due to taste discrimination by shareholders, lenders, input providers, or workers, who dislike interacting with women, especially if they are very talented women (as in Becker, 1981).

From the supply side, this gap may be driven by social or cultural norms that make women devote more time to non-market responsibilities.[13] If talented women are more likely to marry talented men, for example, they may stay out of the labor market after marriage and, hence, we do not observe them as firm managers. At the same time, entrepreneurship is likely to require long hours, especially in the case of large firms, although it also allows for a more flexible schedule than other occupations, especially in the case of small firms.[1414] Given this, one would expect the gender gaps in entrepreneurship rates to be larger at high ability levels. Women with

11 See "Indicadores de la educacion en Chile, 2007-2008".

12 Throughout the text, we assume that more skilled or talented women tend be to more educated.

13 Cubas et al. (2019), for instance, use the American Time Use Survey (ATUS) to document that women have more household care responsibilities than men.

14 Bento et al. (2021) find that female entrepreneurs in the U.S. are much more likely to claim "flexibility of schedule" or "family obligations" as a motive for entrepreneurship. They also show that both male and female employers work longer hours than employees, while self-employed females work shorter hours than employees.

high ability may be less likely to become firm managers because, with some paid household help, they have access to better employment opportunities. Less talented women, on the other hand, are probably not be able to afford the same level of household assistance as more talented ones and have worse employment opportunities. For this reason, these group of women may be more likely to become self-employed or managers of small firms, since this option gives them more time flexibility.

Importantly for us, both demand and supply stories generate a misallocation of talent, since many talented women stay out of entrepreneurship. This generates aggregate losses to the extent that less talented men are then likely to replace them and become the managers of some firms. In the framework presented in the next section we introduce barriers to the occupational choice of women that distort the allocation of talent in the economy. These barriers are aimed at capturing the impediments faced by women to become entrepreneurs, which may be due to explicit discrimination by other agents in the economy or to differences in social norms.

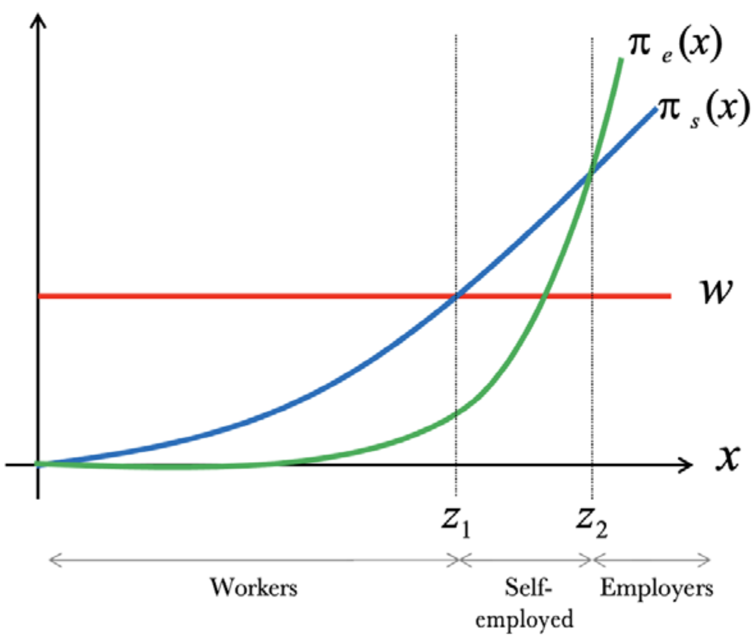
Theoretical framework

Setup description

In order to help interpret the empirical results of Section 2, we present here a brief summary of the model developed in Cuberes and Teignier (2016), while the details of that model can be found in Appendix B. In the next subsection we describe the extension we introduce into the framework to account for the empirical results described above. We use a general equilibrium occupational choice model where agents are endowed with a random entrepreneurship skill, based on which they decide to work as either employers, self-employed, or workers. An employer in this model can produce goods using a span-of-control technology that combines his or her entrepreneurship skills, capital, and workers. This span-of-control element implies that more talented agents run larger firms, as in Lucas (1978). On the other hand, a self-employed agent can produce goods using a similar technology - adjusted by a productivity parameter - but without hiring any workers.

Figure 2 displays the payoff of the three occupations at each talent level and shows that in this model agents with the highest entrepreneurship skill (those with a talent equal or larger than z_2) optimally choose to become employers, whereas those with the least skill become workers (with a talent lower than z_1), leaving the self-employed occupation to agents with intermediate skill levels.

Figure 2. The occupational map



In this economy, aggregate production per capita, which is the sum of output by male employers and self-employed, as well as output by female employers and female self-employed:

$$y \equiv \frac{Y}{N} = \left[\int_{z_2}^{\infty} y(x)d\Gamma(x) + \int_{z_1}^{z_2} \tilde{y}(x)d\Gamma(x) + (1-\theta)\int_{B}^{z_1} \tilde{y}(x)d\Gamma(x)\right]$$

where employers' output is equal to $y(x) = x\, k(x)^{\alpha} n(x)^{(1-\alpha)\,\eta}$ and self-employed workers' output is equal to $\tilde{y}(x) = \tau x \tilde{k}(x)^{\alpha\eta}$. $\Gamma(x)$ denotes the talent cumulative density function and $1-\theta$ is the fraction of the workforce that chooses self-employment because they could not find a job as workers.

To simulate the model, we use a Pareto function for the talent distribution, as in Lucas (1978) and Buera et al. (2011), so the cumulative distribution of talent is given by

$$\Gamma(x) = 1 - B^{\rho}x^{-\rho}, x \geq 0, \text{ where } \rho, B > 0.$$

Introducing gender gaps into the framework

The model assumes that men and women are identical in terms of their innate managerial skills. However, women are subject to some exogenous constraints in the labor market, namely a fraction of women who would like to be employers or self-employed are excluded from these occupations. It is important to note that these constraints are aimed at capturing all barriers, implicit and explicit, that distort the occupation choice of women. Importantly, these restrictions alter the occupational choice and have general equilibrium implications that affect the average talent of entrepreneurs and the aggregate productivity. The intuition behind the output loss caused by the introduction of these gender gaps is as follows. Assume a woman with very good management skills happens to be barred from becoming an employer. The model then implies that a less skilled man will take her position and become the manager of a firm. But note that, if this man has a lower managerial skill than the woman who is not allowed to become a manager, he will run a smaller firm - due to the nature of the span-of-control technology. This would then have general equilibrium implications in terms of the amount of output produced, wages and firms' profits. In particular, it is easy to show that output/income per worker would be lower in this economy as a result of this restriction.

The first constraint we impose is that females face a probability $\mu(x)$ of being allowed to be an employer and a probability $1-\mu(x)$ of being excluded from employership. This probability depends on their ability as follows:

$$\mu(x) = \begin{cases} \tilde{\mu}\left(1 - \left(\frac{x}{B}\right)^{-\gamma}\right) \in [0, \tilde{\mu}] & if\ \gamma > 0 \\ \tilde{\mu}\left(\frac{x}{B}\right)^{\gamma} \in [0, \tilde{\mu}] & if\ \gamma \leq 0 \end{cases}$$

so that

$$\mu'(x) = \begin{cases} \gamma\tilde{\mu}\left(\frac{x}{B}\right)^{-\gamma-1} > 0 & if\ \gamma > 0 \\ \gamma\tilde{\mu}\left(\frac{x}{B}\right)^{\gamma-1} \leq 0 & if\ \gamma \leq 0 \end{cases}$$

Out of the latter group of women, i.e. women not allowed to be employers, some have the possibility of becoming self-employed while the rest are also excluded from self-employment. In particular, women excluded from employership have a probability μ_o of being allowed to be self-employed and a probability $(1-\mu_o)$ of not being allowed to be self-employed. As a result, for each talent level x, a fraction $(1-\mu(x))(1-\mu_o)$ of women are shut out from entrepreneurship, i.e. both employership and self-employment, and can only become workers.[15]

Therefore, the key question to understand the magnitude of the income loss is which type of women are excluded from entrepreneurship. In Cuberes and Teignier (2016), since we do not have firm-level information, we assume that the probability of facing exclusion is independent of the managerial talent. The output loss, however, is definitely lager (lower) if more (less) talented women face a higher probability of being excluded from entrepreneurship since this would imply a bigger drop in the average talent of firm managers. Our analysis in section 2 shows that female-run firms are significantly smaller and less productive than male-run firms, which can be interpreted as evidence that the probability of exclusion is lager for high-talent women. In the next section we parametrize the model to replicate the firm-level Chilean data and obtain quantitative predictions of the efficiency losses.

Numerical results

Model Parametrization

To simulate the model, we first need to give values to its different parameters, as showed in Table 7. The talent distribution function parameter, ρ, and the self-employed productivity penalty parameter, τ, are taken from our paper Cuberes and Teignier (2016), where we calculate the values that allow our framework to match the OECD and ILO data on employers and self-employed share. We choose the value of 0.83 for η, the span of control parameter, to match the share of employers in Chile. This value is slightly larger than 0.79, the one used in Cuberes and Teignier (2016), because it implies managers are able to manage more productive inputs and, hence, it gives a smaller share of employers in equilibrium.

Importantly for this study, we choose the values of the employers' gender gap probability function in equation (3) to match the female firm-size distribution relative to the male one. In calculations not shown here we found that the density of female-run firms is similar to the male one for small firms, suggesting a value μ close to 1. At the same, we also see that the density of female-run firms gets smaller relative to male-run firms as we increase the firm size, which implies a negative value for the parameter γ. To match the female-to-male ratio in the share of employers of approximately 1/3 in the ILO data, we need $\gamma = -1.6$.

Effects from gender gaps

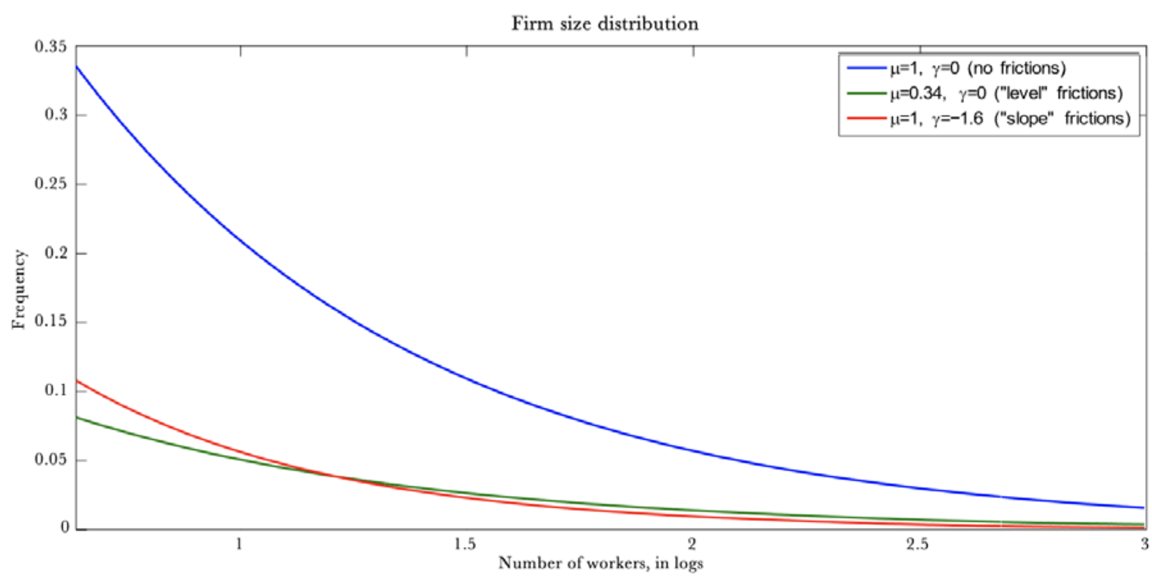
When some women do not participate in employership, the density of female-run firms is below the male- run firms one. This is illustrated in Figure 4, where the lines with positive gender gaps are below the one without gender gaps (i.e. $\mu = 1$ and $\gamma = 0$). We can also see that constrained firm-size distribution is different depending on whether the gender gap is on the intercept μ or the slope γ in equation (3). In the first case, the density reduction is the same at all size levels, while in the second case the density reduction is larger at higher size levels.

15 Note that, in this setup, we are not allowing for the possibility of women being excluded from self-employment but not from employership, since we think that whichever are the barriers women face to become self-employed, they should apply even more strongly to become an employer. In terms of the parameters of the model, if $\mu = 1$, then the value of μ_o does not affect the occupational choices of women.

Table 7. Parameter values

Parameter	Value	Explanation
B	1	Normalization
η	0.83	To match share of employers in Chile
a	0.154	To match capital share $a\eta + (1-\eta) = 0.3$
ρ	6.5	From Cuberes and Teignier (2016)
τ	0.7	From Cuberes and Teignier (2016)
μ	1	To match women firm size distribution relative to men
γ	1.6	To match women firm size distribution relative to men

Figure 3. Gender gaps effects on firm-size distribution



When we take into account that more talented females face a higher probability of not participating into employership, i.e. the last column of Table 8, we calculate an output loss of 7.5%. This compares to a loss of 5.2% in the case of random exclusion, i.e. column 2 of this table. This larger loss is due to the fact that the average talent of employers falls more, as the lower value of z_2 illustrates. This leads to a larger fall in the aggregate productivity economy, which also implies a deeper fall in the wage rate w. The average talent of self-employed also falls more, since both thresholds z_1 and z_2 fall more, which also contributes to the larger output fall in the last column. When more talented females face a higher probability of not participating into employership, the average talent of female employers is lower than the male employers one, which leads to the average employers' earnings gap observed in the previous to last row of Table 8.

Table 8. Quantitative effects of gender gaps

	Random exclusion $\tilde{\mu} = 0.34$, $\gamma = 0$	Exclusion increasing with talent $\tilde{\mu} = 1$, $\gamma = -1.6$
ω	0.9	0.89
z_1	1.59	1.57
z_2	1.75	1.72
Employers earnings gap	0%	61%
Output loss	**5.21%**	**7.53%**

Conclusions

This paper uses firm-level data from the Chilean Encuestas Longitudinales de Empresas for the year 2007 to document the differences between male and female-run firms. We find that only about one third of the firms are run by women and that female-run firms are significantly smaller and less productive than the male-run ones, even after controlling for the age, education, and experience of the manager as well as the firm sector and ownership structure. Moreover, the entrepreneurship gender gap tends to increase with the education level of the firm manager. We extend the framework developed in Cuberes and Teignier (2016) to replicate these findings and conclude that gender gaps in entrepreneurship in Chile generate a fall in aggregate productivity and aggregate income of 7.5%. This is significantly higher than the income loss predicted by a model without gender-differences in firm size.

As discussed above, through the lens of our model, our interpretation of these facts is that the implicit barriers faced by women to become entrepreneurs are larger for more skilled women than for less skilled women. This leads to a negative selection of women into entrepreneurship, which we think would be interesting to further explore in the future. More research is clearly needed to confirm this interpretation as well as the economic forces behind them. We concur with other articles in the literature that the combination of long hours and schedule flexibility in entrepreneurship are important factors behind the observed patterns.

References

Arellano, P, and Peralta, S. (2015). "Informe de resultados: análisis de género en las empresas Tercera Encuesta Longitudinal de Empresas." Ministerio de Economia, Fomento y Turismo. División de Política Comercial e Industrial Agosto.

Becker, G. S. (1981). A treatise on the family. Cambridge, Mass: Harvard University Press.

Bento, P. (2021). "Female Entrepreneurship in the U.S. 1982-2012: Implications for Welfare and Aggregate Output." Working Papers 20211108-001, Texas AM University, Department of Economics.

Bento, P., L. Shao, and F. Sohail (2021). "Gender Gaps in Time Use and Entrepreneurship." Manuscript.

Bertrand, M. (2017), "The Glass Ceiling." Becker Friedman Institute for Research in Economics Working Paper No. 2018-38.

Cameron, A. C., and Trivedi, P. K. (2005). Microeconometrics: Methods and Applications. Cambridge.

Cavalcanti, T., and Tavares, J. (2016). "The Output Cost of Gender Discrimination: A Model-Based Macroeconomic Estimate." Volume 126, Issue 590, February, pp. 109–134.

Chiplunkar, G., and P. K. Goldberg (2021). "Aggregate Implications of Barriers to Female Entrepreneur- ship," NBER Working Papers 28486.

Crespi, G., Tacsir, E., and Vargas, F. (2016). "Innovation Dynamics and Productivity: Evidence for Latin America." Chapter 2 in Firm Innovation and Productivity in Latin America and the Caribbean. Inter-American Development Bank.

Cubas, G., C. Juhn, and P. Silos (2019). "Coordinated Work Schedules and the Gender Wage Gap." National Bureau of Economic Research, Inc NBER Working Papers 26548.

Cubas, G., C. Juhn, and P. Silos, "Work-Care Balance Over the Day and the Gender Wage Gap." Forthcoming, American Economic Review PP.

Cuberes, D., and Teignier, M. (2016). "Aggregate Costs of Gender Gaps in the Labor Market: A Quantitative Estimate." Journal of Human Capital, vol. 10, no. 1.

Cuberes, D., and Teignier, M. (2014). "Gender Inequality and Economic Growth: A Critical Review." Journal of International Development, vol. 26, Issue 2, pp. 260-276, March.

Dayıoğlu, M., and Kırdar, M.G. (2010). "Determinants of and Trends in Labor Force Participation of Women in Turkey." Welfare and Social Policy Analytical Work Program Working Paper Number:5. State Planning Organization of the Republic of Turkey and World Bank.

Dollar D., and Gatti, R. (1999). "Gender Inequality, Income and Growth: Are Good Times Good for Women?" Policy Research Report on Gender and Development Working Paper Series No. 1. World Bank, Washington, DC.

Galor, O., and Weil, D. N. (1996). "The Gender Gap, Fertility, and Growth." American Economic Review 85(3), 374–387.

Goldin, C., "A Grand Gender Convergence: Its Last Chapter," American Economic Review, 2014, 104 (4), 1091–1119

Greene, W. H. (2012). Econometric Analysis. Seventh Edition. Pearson.

Hsieh, C. T., Hurst, E., Jones, C. I., Klenow, P.J. (2019). The allocation of talent and US economic growth. Econometrica, 87(5), 1439-1474.

International Labor Organization (2104). "Global Employment Trends."

Kantis, H., Federico, J., Angelelli, P, and Garcia, S.I. (2016). "Business Performance in Young Latin American Firms." Chapter 6 in Firm Innovation and Productivity in Latin America and the Caribbean. Inter-American Development Bank.

Lucas Jr., R. E. (1978). "On the Size Distribution of Business Firms." The Bell Journal of Economics 9(2), 508-523.

MANUAL DE USO Encuesta Longitudinal de Empresas-2007, Gobierno de Chile.

Morazzoni, M., and A. Sy (2021). "Female Entrepreneurship, Financial Frictions and Capital Misallo- cation in the US." Manuscript.

Ranasinghe, A. (2020). "Misallocation across Establishment Gender," Working Papers 2020-2, Univer- sity of Alberta, Department of Economics.

Variyam, J. N., and Kraybill, D. S. (1992). "Empirical Evidence on Determinants of Firm Growth," Economics Letters, Volume 38, Issue 1, January, pages 31-36.

World Bank, (2012). "World Development Report 2012: Gender Equality and Development."

Appendix

Robustness analysis

Out-of-necessity entrepreneurs

It is often argued that in many developing countries individuals choose to become self-employed (or even entrepreneurs) out of necessity. In other words, while they will be better off working for someone else, they cannot find such an occupation. [16]

16 Using data from the Global Entrepreneurship Monitor survey, Poschke (2013) fins that necessity entrepreneurs represent almost 50% of all entrepreneurs in non-OECD countries.

To address this possibility, we re-estimate the regressions of firm size dropping from the sample the smallest 5% firms. The rationale for this is that it is often the case that very small firms are run by out- of-necessity employers. The results, displayed in Tables suggest that dropping the 5% smallest firms has no significant impact on the coefficient associated with men, the key variable in our analysis.

Table 9. Determinants of firms' size (excluding 5% smallest firms) using *tamaño6*

	(1)	(2)	(3)	(4)	(5)
Men	0.53***	0.47***	0.44***	0.45***	0.36***
Age	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
Experience			-0.03	-0.002	-0.004**
			(0.002)	(0.002)	(0.002)
			0.005***	0.005***	0.006***
			(0.002)	(0.002	(0.002)
Education dummies	No	Yes	Yes	Yes	Yes
Sector dummies	No	No	No	Yes	Yes
Type of firm	No	No	No	No	Yes
Pseudo R^2	0.01	0.04	0.04	0.05	0.11
N	7249	7249	7244	7244	7244

Model Details

The economy we consider has a continuum of agents indexed by their entrepreneurial talent x, drawn from a cumulative distribution Γ that takes values between B and ∞. We assume the economy is closed and that it has a workforce of size N and K units of capital. Labor and capital are inelastically supplied in the market by consumers, in exchange for a wage rate w and a capital rental rate r respectively. These inputs are then combined by firms to produce an homogeneous good. Agents decide to become either firm workers, who earn the equilibrium wage rate w – which we assume to be independent of their entrepreneurial talent–, or entrepreneurs, who earn the profits generated by the firm they manage. In what follows we will refer to an entrepreneur as someone who works as either an employer or a self-employed. In the model simulation, we also include a fourth category, namely the out-of-necessity entrepreneurs, who choose this occupation because they had no other occupational choices apart from running their own business. We denote by $1-\theta$ the fraction of both males and females that are out-of-necessity entrepreneurs.

The agents' optimization problem and occupation map in this version of the model is exactly the same as the one discussed in Section 3. However, the market-clearing conditions are now different to reflect the new restrictions in the labor market.

An agent with entrepreneurial talent or productivity level x who chooses to become an employer and hires $n(x)$ units of labor and $k(x)$ units of capital produces $y(x)$ units of output and earns profits $\pi(x) = y(x) - rk(x) - wn(x)$, where the price of the homogeneous good is normalized to one. As in Lucas (1978) and Buera and Shin (2011), the production function is given by

$$y(x) = x(k(x)^{\alpha} n(x)^{1-\alpha})^{\eta},$$

where $\alpha \in (0,1)$and $\eta \in (0,1)$. The parameter η measures the span of control of entrepreneurs and, since it is smaller than one, the entrepreneurial technology involves an element of diminishing returns. On the other hand, an agent with talent x who chooses to become self-employed uses the amount of capital $\tilde{k}(x)$, produces $\tilde{y}(x)$ units of output and earns profits $\tilde{\pi}(x) = \tilde{y}(x) - r\tilde{k}(x)$. The technology she operates is

$$\tilde{y}(x) = \tau x \tilde{k}(x)^{\alpha\eta},$$

where τ is the self-employed productivity parameter. The consumption good produced by the self-employed and the capital they use is the same as the one in the employers' problem. However, it is convenient to denote them $\tilde{y}$ and $\tilde{k}$ to clarify the exposition. One interpretation of this parameter is that self-employed workers have to spend a fraction of their time on management tasks, which would imply that τ is equal to the fraction of time available for work to the power $(1 - \alpha)\eta$. As explained below, we estimate this parameter to match the average fraction of self-employed in the data.

Agents' optimization

Employers

Employers choose the units of labor and capital they hire in order to maximize their current profits π. The optimal number of workers and capital stock, $n(x)$ and $k(x)$ respectively, depend positively on the productivity level x, as equations (6) and (7) show:

$$n(x) = \left[x\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{1/(1-\eta)}$$

$$k(x) = \left[x\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{1/(1-\eta)}.$$

Self-employed

When we solve for the problem of a self-employed agent with talent x who wishes to maximize his or her profits, we find

$$\tilde{k}(x) = \left(\frac{\tau x\alpha\eta}{r}\right)^{\frac{1}{1-\alpha\eta}}.$$

Occupational choice

Figure 2 displays the shape of the profit functions of employers ($\pi_e(x)$) and self-employed ($\pi_s(x)$) along with wage earned by workers as a function of talent x. In order to construct this figure, we are implicitly using values for the parameters τ, α, and η, such that the three occupations are chosen in equilibrium. The figure also shows the relevant talent cutoffs for the occupational choices Here we present the equations that define these thresholds: the first one, z_1, defines the earnings such that agents are indifferent between becoming workers or self-employed and it is given by

$$w = \tau z_1 \tilde{k}(z_1)^{\alpha\eta} - r\tilde{k}(z_1)$$

If $x \le z_1$ agents choose to become workers, while if $x > z_1$ they become self-employed or employers. The second cutoff, z_2, determines the choice between being a self-employed or an employer and it is given by

$$\tau z_2 \tilde{k}(z_2)^{\alpha\eta} - r\tilde{k}(z_2) = z_2 x(k(z_2)^{\alpha} n(z_2)^{1-\alpha})^{\eta} - rk(z_2) - wn(z_2)$$

so that if $x > z_2$ an agent wants to become an employer.

Competitive Equilibrium

We assume that women represent half of the population in the economy and that there is no unemployment. Moreover, any agent in the economy can potentially participate in the labor market, except for the restrictions on women described above. Under these assumptions, in equilibrium, the total demand of capital by employers and self-employed must be equal to the aggregate capital endowment (in per capita terms) k:

$$\begin{aligned} k = &\frac{1}{2}\left[\int_{z_2}^{\infty} k(x)d\Gamma(x) + \int_{z_1}^{z_2} \tilde{k}(x)d\Gamma(x) + (1-\theta)\int_{z_1}^{z_2} \tilde{k}(x)d\Gamma(x)\right] \\ &+\frac{\lambda}{2}\left[\int_{z_2}^{\infty} \mu(x)k(x)d\Gamma(x) + \int_{z_1}^{z_2} \big(\mu(x) + (1-\mu(x))\mu_0\big)\tilde{k}(x)d\Gamma(x)\right] \\ &+\frac{\lambda}{2}\left[\int_{z_2}^{\infty} (1-\mu(x))\mu_0\tilde{k}(x)d\Gamma(x) + (1-\theta)\int_{B}^{z_1} (\mu(x) + (1-\mu(x))\mu_0)\tilde{k}(x)d\Gamma(x)\right] \end{aligned}$$

The upper term is the demand for capital by men and the two lower terms are the women's demand for capital. The demand for capital by male-run firms has three components: the first one represents the capital demand by employers, while the second and third terms represent the demand by self-employed. i.e. those who have the right ability to be self-employed plus the capital demand by those who become self-employed because they do could not find a job as workers. As explained in Section 3, a fraction $(1 - \theta)$ of both males and females with ability below z_1 become self-employed because they would like to be workers but are not allowed to do so and choose their second-best option. These out-of-necessity self-employed demand the optimal amount of capital given their talent or ability.

The demand of capital by female-run firms has four components, each of them multiplied by the fraction of women in the labor force, $\frac{\lambda}{2}$. The first one represents the capital demand by female employers, i.e. those with enough ability to be employers and who are allowed to be so, while the second term represent the capital demand by women who have the right ability to be self-employed and are allowed to work. The third term shows the capital demand by women who become self-employed because they are excluded from employership and, finally, the last term shows the fraction of females who would like to be workers but, since they are "excluded" from this occupation, they choose to become out-of-necessity self-employed if they are not excluded from entrepreneurship.[17] Similarly, the labor market-clearing condition is given by

$$\frac{1}{2}\left[\int_{z_2}^{\infty} n(x)d\Gamma(x))\right] + \frac{\lambda}{2}\int_{z_2}^{\infty} \mu(x)n(x)d\Gamma(x) = \frac{1}{2}\theta\Gamma(z_1) + \frac{\lambda}{2}\theta\left[\Gamma(z_1) + \int_{z_1}^{\infty} \big((1-\mu(x))(1-\mu_0)\big)d\Gamma(x)\right]$$

where the first line represents the aggregate labor demand and the second line represents the aggregate labor supply. The first term is the labor demand by male employers and the second one corresponds to the labor demand by female employers, i.e. those women with enough ability to be employers who are allowed to choose their occupation freely. The first term of the labor supply shows the fraction of men who choose to become workers, while the second and third show the fraction of female workers. The latter terms is the fraction of females who, given their talent want to be workers as well as the fraction of females who have enough ability to be

17 Note that this setup implies that, for each talent level x, a fraction $(1-\theta)\big(1-\mu(x)\big)\big(1-\mu_o\big)$ are excluded from all employment categories and, hence, are forced out the labor force.

employers or self-employed but are excluded from both occupations. For these group of women, the only option is to try to become workers.

A competitive equilibrium in this economy is a pair of cutoff levels (z_1, z_2), a set of quantities $[n(x), k(x), \tilde{k}(x)], \forall x$, and prices (w, r) such that entrepreneurs choose the amount of capital and labor to maximize their profits, and labor and capital markets clear.